

Overview, Goals of Longitudinal Research, and Historical Developments Juliford Pres

OVERVIEW

This book is written with the intent to lead readers from the basics of growth modeling to several advanced topics including growth mixture models, nonlinear change models, growth models for non-normal outcomes, growth models of latent variables, and the recent advances in latent change score modeling. In its entirety, the book is meant to support graduate courses on longitudinal data analysis and latent growth modeling in the social, educational, and behavioral sciences, or researchers interested in incorporating these methods into their research programs.

The 18 chapters are organized into five parts. In the first part, Introduction and Organization, we review the goals of longitudinal research and some practical preliminary steps that should be taken prior to examining change (descriptive statistics and plotting of longitudinal data). In the second part, The Linear Growth Model and Its Extensions, we introduce the linear growth model and several ways to expand the model to examine between-person differences in linear change and study multivariate change. Specifically, we cover different ways to handle time, the inclusion of time-invariant covariates as predictors of the growth factors (intercept and slope), multiple-group growth models, growth mixture models, and multivariate growth models. Several advanced topics are introduced in this part but are presented in the context of the linear growth model. The third part, Nonlinearity in Growth Modeling, proceeds through an array of nonlinear models—growth models that are nonlinear with respect to time, growth models that are nonlinear with respect to parameters, and growth models that are nonlinear with respect to random coefficients (latent variables). The stepwise presentation is organized to facilitate adoption of increasingly complex models. The fourth part, Modeling Change with Latent Entities, addresses the application of growth models that are fit directly to binary (dichotomous) and ordered polytomous outcomes, and latent variables that are indicated by multiple continuous and ordinal variables. The fifth and final part, *Latent Change Scores as a Framework for Studying Change*, introduces a process-oriented version of the growth model. We discuss univariate and bivariate models, and then describe recent advancements in how these models can be used to study individual rates of change in nonlinear growth models.

With intent to facilitate practical application of growth models to longitudinal data, all of the models are introduced with detailed presentation of real-data examples, code for fitting the models to the example data using multiple statistical packages, discussion of the output from those programs, and interpretation of the modeling results. Remaining agnostic to the modeling framework, we introduce each topic through the multilevel and structural equation modeling frameworks. Within the multilevel modeling framework we provide code for PROC NLMIXED in SAS (Littell, Milliken, Stroup, Wolfinger, & Schabenberger, 2006) and the nlme package (Pinheiro, Bates, DebRoy, Sarkar, & R Development Core Team, 2013) in R. Within the structural equation modeling framework we provide code for Mplus (Muthén & Muthén, 1998–2012) and the OpenMx package (Boker et al., 2011) in R. In each framework, we have purposively paired a popular proprietary program (SAS and Mplus) with a freely available R package (nlme and OpenMx) so that all readers will be able to work through the examples in at least two programs. Of practical note, we have utilized the nonlinear mixed-effects modeling programs (NLMIXED and nlme) instead of their associated linear mixed-effects modeling programs (MIXED and 1me) because the nonlinear programs are more flexible and therefore can be used to fit more of the models we present. Additionally, the programming of these procedures closely follow the mathematical presentations of the models, which we feel aids understanding. Finally, we provide code for the linear mixed-effects modeling programs on our website.

FIVE RATIONALES FOR LONGITUDINAL RESEARCH

In working through the chapters, it may be useful to keep in mind specific research questions and how the longitudinal data being analyzed help to propel those questions. In the dialectic surrounding lifespan development in the 1970s, Baltes and Nesselroade (1979) outlined five main rationales for conducting longitudinal research. At the time, these rationales described opportunities that longitudinal research designs afforded and laid the groundwork and impetus for the development of new methods to analyze longitudinal data. Growth models can be viewed, in part, as an answer to the call—these methods provided a statistically rigorous framework that enabled researchers to take advantage of the opportunities brought about by the collection of longitudinal data (see McArdle & Nesselroade, 2014). In the chapters that follow we often refer back to Baltes and Nesselroade's five rationales, and thus present them here, at the outset, as an overarching framework within which to consider one's research goals.

• **Rationale 1.** The first rationale and primary reason for conducting longitudinal research is the *direct identification of intraindividual change (and stability)*. Measuring the same individual (entity) repeatedly allows researchers to identify if and how specific

attributes of the individual changed (or remained the same) over time. Developmental (and other) theories of change often conceptualize and describe change as either an incremental or a transformational process (see Ram & Grimm, 2015). Incremental change is observed and identified as change in the magnitude (quantitative) of the same construct along a continuum over a specific time interval. Transformational change is observed and identified as a change or transition between discrete states during a specific time interval (e.g., Piaget's stage theory of development; Piaget, 1952). Analytically, the main goal is to obtain a parsimonious and accurate description of how and when attributes of the individual change over time. Importantly, Baltes and Nesselroade noted that stability and constancy over time are special cases of intraindividual change. As we shall see in the rest of the book, growth models are designed specifically to articulate a wide variety of possible (linear and nonlinear) patterns of intraindividual change.

- Rationale 2. Once the pattern of intraindividual (within-person) change is identified (in terms of magnitude or sequential steps), a logical next question to ask is whether different individuals change in different ways. Thus, the second rationale for longitudinal research is the *direct identification of interindividual differences* (or similarity) in intraindividual change. This rationale invokes research questions like Do different individuals change different amounts or in different directions?, or Do different individuals transition from one stage to another at different times? Baltes and Nesselroade (1979) suggested that heterogeneity in change is the norm given the "existence of diversity, multidirectionality, and large interindividual differences in developmental outcomes" (p. 24). As discussed in Chapters 3, 5, 6, and 7, growth models are structured specifically to describe interindividual differences in intraindividual change.
- Rationale 3. Acknowledging that change rarely occurs in isolation, the third rationale for longitudinal research is the *analysis of interrelationships in behavioral change*. As Baltes and Nesselroade (1979) note, "The examination of interrelationships in change among distinct behavioral classes is particularly important if a structural, holistic approach to development is taken" (p. 25). This holistic approach centers on the idea that changes in multiple constructs are expected to occur simultaneously and/or sequentially. Analytically, the task requires simultaneous analysis of multiple variables and the evaluation of how changes in one variable precede, covary, and/or follow changes in another variable. In Chapter 8 we discuss multivariate growth models and dynamic predictors, and in Chapter 17 we cover how latent change score models may be used to examine such interrelationships.
- **Rationale 4.** The fourth rationale, *analysis of causes (determinants) of intraindividual change*, centers on explaining or accounting for the observed within-person change process. Specifically, the objective is to identify the time-varying factors and/or mechansims that impact and/or drive the within-person changes identified in Rationale 1. Key in our presentation is that changes are likely to proceed at different rates at different periods of time. For example, when learning a new skill, intraindividual changes may proceed quickly early on, but more slowly later as individuals reach asymptotic levels of

performance. In Chapter 8 we cover how time-varying predictors can be introduced into the growth model, and later in Part III (Chapters 9 to 12), we address nonlinear models for intraindividual change.

• Rationale 5. The fifth rationale for longitudinal research is the *analysis of causes* (*determinants*) of interindividual differences in intraindividual change. Given that individuals differ in how they change over time (Rationale 2), researchers are often interested in identifying the factors and/or mechanisms that can account for those between-person differences. The objective is to identify the time-invariant variables that are related to specific aspects of within-person change. For example, demographic/background characteristics, experimental manipulations (e.g., interventions), and characteristics of the individuals' proximal and distal contexts may all influence how and when change proceeds. Research questions proceeding from Rationale 5 are often examined through the inclusion of time-invariant covariates (Chapter 5), the use of multiple-group growth models (Chapter 6), and growth mixture models (Chapter 7).

Together, these five rationales for longitudinal research provide the foundation for building precise research questions that can be examined using contemporary growth models and the extensions covered in this book. As you work through the chapters, we encourage you to articulate how your research paradigms map on to these rationales. What is your theory of intraindividual change? What is your theory of between-person differences? and so on. You can then select specific models that are appropriate for those questions, and you can thoughtfully consider if and how the data afford and/or limit your ability to obtain accurate answers.

HISTORICAL DEVELOPMENT OF GROWTH MODELS

Before proceeding to the specifics of contemporary growth models and their recent extensions, we discuss the historical context in which growth models were developed. The methods we use to analyze change emerged from almost a century's worth of innovations. This summary provides a brief and selective overview of the innovations that contributed to the models presented throughout this book.

The beginning of growth modeling and the ideas underlying many of the methods used today can be traced back to Wishart's (1938) critique of a study examining the weight gain of three groups of bacon pigs that were on three different diets (Woodman, Evans, Callow, & Wishart, 1936). Woodman et al. (1936) had calculated each pig's overall weight gain as the difference between the pig's weight at baseline and at week 16, and used the resulting change scores as the dependent variable in an analysis of variance to examine differences in weight gain in relation to diet type. The results were lackluster, with no significant differences in total weight gain between the three diet groups. Discouraged, but persistent, the authors then conducted an analysis of covariance that included baseline weight as a covariate. This analysis supported the initial hypothesis and

provided evidence of a significant difference in weight gain between two of the three diet groups. Wishart (1938) was concerned, not with the soundness of the statistical analysis, which were indeed proper, but with the extent of *unanalyzed data*. The weights of the pigs were recorded weekly. However, the analysis only used the measurements obtained at baseline (week 0) and week 16. The original analysis used only those data that would conform to a straightforward analysis of variance and covariance. That is, the researchers selected data that fit into a specific analytic technique, rather than utilizing all of the data that were collected. Wishart (1938) thought that analyzing all 17 repeated observations would yield a more reliable and valid answer to the research question Do pigs' diets impact their rates of growth? The predicament was that it was not yet clear how all the repeated measures could be used to track the within-pig changes and the between-pig differences in within-pig change.

In his critique, Wishart (1938) approximated the formal methods that would be developed 50 years later. Following good practices, he first plotted the data—pigs' weight and the log transform of the pigs' weight on the y-axis and time (weeks since the beginning of the study from 0 through 16) on the x-axis. Then, examining these plots, he sought to identify a mathematical function that would provide the best representation of each pig's growth trajectory. After considering a few options, Wishart decided on a quadratic polynomial of the form $y_1 = b_1 + b_2 \cdot (t - 8) + b_3 \cdot \{(t - 8)^2 - 24\}$ and estimated the parameters of the quadratic curve (i.e., b_1 , b_2 , and b_3) that best described each pig's data. These included an intercept (centered at week 8), a linear change component interpreted as "average growth rate in pounds per week," and a quadratic change component interpreted as "half the rate of change in the growth rate in pounds per week" (i.e., a scaling of acceleration). Thus, Wishart reduced the dimensionality of the original data (17 repeated measures) down to three specific aspects of growth that he thought had substantive meaning and that, hopefully, sufficiently described the entirety of the growth process. Wishart then used an analysis of variance to determine whether differences in the pigs' "average growth rate" (linear component) were related to diet. Wishart found a significant difference between two of the three diets in the linear aspect of change. As with the original analysis, Wishart then conducted an analysis of covariance accounting for the pigs' initial weights (specifically, predicted initial weight from the individual quadratic models). Replicating the original results, he found significant differences in "average growth rate" between two of the three diet groups. He then conducted similar analyses for the 'rate of change in the growth rate' (quadratic component). Wishart found that the three diet groups differed significantly in how their rate of weight gain accelerated over time.

Overall, Wishart's results were more robust (results were stronger) when using all of the longitudinal data, and he attempted to capture multiple aspects of the change process. Wishart's point was that there was important information embedded in *all* of the repeated measures and that information could be used to provide more accurate descriptions of the within-pig change process and the between-pig differences in the within-pig change process. The density of the repeated measures provided a more complex representation of growth and a better understanding of the growth process.

The general approach that Wishart used provides the foundation for understanding the core aspects of contemporary growth models. Key aspects of Wishart's approach were that (1) an individual's observed change trajectory can be described by a mathematical function of time, plus noise (error), (2) the parameters of the function represent specific, meaningful aspects of the within-individual change process (Rationale 1), (3) variation in those parameters constitutes information about between-individual differences in the change process (Rationale 2), and (4) how the variation in the growth parameters can be associated with other predictor variables or covariates provides information about exogenous (diet) and endogenous (initial weight) determinants of the between-individual differences in the within-individual change process (Rationale 5). The utility of Wishart's approach prevails today. Initial steps in the study of individual change often include plotting individual trajectories and fitting individual regressions to estimate individual growth parameters (see Singer & Willett, 2003).

Twenty years after Wishart's analysis, Tucker (1958) and Rao (1958) presented work that is often cited as the foundation of growth models within the structural equation modeling framework. Rao and Tucker each proposed an approach wherein the sums of squares and cross-products matrix obtained from repeated measures data were subjected to a principal components analysis. The principal components model decomposed the repeated measures data into a set of generalized learning curves, component loadings representing distinct patterns of change, and individual component weights (component scores) indicating the degree to which an individual's observed trajectory was saturated by each of the generalized learning curves (components). The generalized learning curves were interpreted as the fundamental aspects of change that all individuals shared (Rationale 1), and the individual component weights indicated how individual trajectories were different from one another (Rationale 2). Tucker (1966) subsequently refined the techniques for determining the number of generalized learning curves (components) to retain and described rotation procedures that would aid interpretation of the learning curves. In the same way that Wishart used a specific mathematical function (quadratic polynomial) to reduce the 17 repeated measurements of a pig's weight down to three meaningful parameters (intercept, rate of change, rate of acceleration) and examined between-pig differences in those parameters, Tucker and Rao used principal components analysis to reduce the dimensionality of the repeated measures data obtained from multiple individuals down to a smaller number of learning curves and examined between-person differences in the weighting of those curves/components. Key links to the application of growth models fit in the structural equation modeling framework are the use of a multivariate approach (i.e., factor-analytic) to reduce dimensionality, the way component (factor) loadings represent the dominant change trajectories, and the use of component (factor/latent variable) scores to provide information about between-person differences in change (see Grimm, Steele, Ram, & Nesselroade, 2013).

Through the 1970s and into the early 1980s the individual growth modeling (from Wishart) and generalized learning curve (from Tucker) approaches were used to examine how individuals changed over time. Of course, estimation routines were updated along the way, with the facility afforded by least squares, nonlinear least squares, and Bayesian

approaches to estimating growth parameters (see Berkey, 1982; Box, 1950; Potthoff & Roy, 1964; Rogosa, Brandt, & Zimowski, 1982). Then, Harville (1977) introduced a class of linear mixed-effects models, and Laird and Ware (1982) developed more efficient estimation techniques for those models (see also Rao, 1965), which provided the main foundations that would support the fitting of growth models in the multilevel modeling framework. Specifically, Laird and Ware (1982) proposed that two-stage models should be used to study change. Using repeated measures of pulmonary function, they demonstrated how the new, unified approach to estimation (simultaneous estimation of level-1 [within-person] and level-2 [between-person] model parameters) could be used to study between-person differences in within-person change (Rationale 2). Further, their demonstration showed how exposure to air pollution had an effect on the long-term development of pulmonary function and highlighted how this framework could handle incomplete and highly unbalanced data—a common feature of longitudinal data. In the years that followed, Rogosa and Willett (1985) and Bryk and Raudenbush (1987) refined how the mixed-effects framework could be used to study individual change. These works highlighted common misconceptions regarding the study of change, demystified how the models articulated theory about individuals' initial state and rates of change (and the assumptions therein), and outlined a variety of change trajectories, linear and nonlinear, that could be examined using the mixed-effects modeling framework. Their presentations of accessible examples prompted many psychologists and educational researchers to adopt these techniques and made them a central part of the statistical toolbox used by social scientists.

In parallel, Jöreskog and Sörbom (1979) developed the structural equation modeling framework and supplied the research community with accessible software that provided the facility for simultaneously modeling mean and covariance structures. Using this framework and giving a nod to the approach introduced by Tucker (1958) and Rao (1958), Meredith and Tisak (1984, 1990) provided a general framework for fitting latent curve models in the structural equation modeling framework. Specifically, they illustrated how the linear growth model can be specified as a restricted confirmatory factor model with a mean structure, and discussed extensions to multiple-group growth models, higher-order polynomial models, spline models, and a variety of models with nonlinear change patterns. The flexibility of the structural equation modeling framework immediately enabled researchers to extend Meredith and Tisak's (1984, 1990) work. In the 1980s, for example, McArdle (1986) combined additive genetic models and latent growth models in the analysis of longitudinal data from twins to assess the additive genetic (heritability), common environmental, and unique environmental components of initial test performance, change in performance over time, and unique (individual) variability. McArdle (1988) also extended the model into the multivariate space, proposing several ways in which growth models could be used to study the development of two or more processes as well as changes in latent variables. The first of these models was the bivariate (or parallel process) growth model where the changes in two variables are simultaneously examined and the associations between intercepts and slopes are evaluated to study whether individual changes in one process are associated with individual changes in the second process. The second model was the curve of factors model or second-order growth model (Hancock, Kuo, & Lawrence, 2001), where changes in a multiply indicated latent variable were modeled. The third model was the factor of curves model where the associations among growth factors (as in the bivariate growth model) were modeled with second-order factors instead of covariance paths. The introduction of these models spurred discussions of how to test whether the same construct was measured in the same scale over time (longitudinal measurement invariance) and how to study the interplay between multiple developmental processes.

As the advances in computational power and efficiency increased, the possibilities for estimating nonlinear mixed-effects models were greatly enhanced (see Davidian & Giltinan, 1995; Pinheiro & Bates, 1995; Vonesh & Chinchilli, 1996). This allowed for the examination of interindividual differences in a wider set of within-person change models in the multilevel modeling framework. Work on this topic was conducted by Lindstrom and Bates (1990), Burchinal and Appelbaum (1991), Beal and Sheiner (1992), Vonesh (1992a, 1992b), Wolfinger (1993), Lin (Wolfinger & Lin, 1997), and Davidian and Gallant (1993). In the structural equation modeling framework, work on this topic was conducted by Browne and du Toit (1991; see also Browne, 1993), who showed how complex nonlinear mixed-effects models could be approximated through Taylor series expansion following the work of Beal and Sheiner (1982). This opened new opportunities to merge the flexibility of the structural equation modeling framework (e.g., measurement models) with the study of inherently nonlinear trajectories (see Blozis, 2004; Grimm, Ram, & Estabrook, 2010).

In the midst of these innovations, the growth modelers working in the multilevel framework (also called mixed-effects or random coefficient models) and the growth modelers working in the structural equation modeling framework realized that the two frameworks could be used to fit the same model and obtain identical results (see Willett & Sayer, 1994). In this book we present the multilevel and structural equation approaches and note that the choice of modeling framework is mostly a matter of preference because nearly all of the models we present can be fit in both frameworks. However, certain models are easier to specify and estimate in one framework versus the other. For example, the mixed-effects modeling framework handles individually varying time scales and modeling of inherently nonlinear trajectories more easily than the structural equation modeling framework, whereas the structural equation modeling framework provides more flexibility into modeling residual structures, fitting multivariate change models, and incorporating multiply indicated latent variables (see Ghisletta & Lindenberger, 2003), although these differences have been minimized over time (Grimm & Widaman, 2010; Kwok, West, & Green, 2007; Sterba, 2014).

Around the turn of the century, there was an increased interest in considering qualitative differences in within-person change (e.g., Magnusson, 2003). Researchers needing facility to group individuals based on their change patterns (e.g., early learners, late learners) introduced semiparametric group-based models, that represented between-person differences in change as a collection of latent classes (Jones, Nagin, & Roeder, 2001; Nagin, 1999), and growth mixture models that represented between-person differences

in change as a combination of latent classes *and* continuous between-person differences within each latent class. Despite some limitations and ambiguity in their use (Bauer & Curran, 2003; Grimm, Ram, Shiyko, & Lo, 2013; Ram, Grimm, Gatzke-Kopp, & Molenaar, 2011), the popularity of these models has produced a great deal of knowledge about how individuals differ in how they change and prompted a rich set of advanced modeling possibilities (see Grimm & Ram, 2009; Grimm, Ram, & Estabrook, 2010; Li, Duncan, Duncan, & Hops, 2001; Ram & Grimm, 2009).

In the 2000s there were also innovations in how growth models could be used to simultaneously model individual changes and examine time-dependent lead-lag associations with longitudinal panel data. McArdle and Hamagami (2001) showed how latent difference (change) variables could be specified through fixed structural paths in the structural equation modeling framework—an extension that allowed researchers to examine the interplay between changes in two or more variables. At the same time, Curran and Bollen (2001) highlighted how autoregressive and cross-lagged effects could be included directly in growth models specified in the structural equation modeling framework. These efforts subsequently led to second-order difference models (Hamagami & McArdle, 2007) to study acceleration and its determinants and latent differential models (Boker, Neale, & Rausch, 2004), which treat time continuously instead of discretely, multiple-group and growth mixture models to examine group differences in lead-lag associations (Ferrer et al., 2007; Grimm, 2006), and the examination of between-person differences in the rate of change in nonlinear models (Grimm, Castro-Schilo, & Davoudzadeh, 2013; Grimm, Zhang, Hamagami, & Mazzocco, 2013). The latent change score framework allows for the examination of all of Baltes and Nesselroade's rationales for longitudinal research (see McArdle, 2009; McArdle & Nesselroade, 2014).

MODELING FRAMEWORKS AND PROGRAMS

As mentioned, we discuss both the structural equation modeling and multilevel modeling frameworks for specifying and fitting growth models. The majority of growth models can be specified in both frameworks (see Curran, 2003; Ghisletta & Lindenberger, 2003; Willett & Sayer, 1994); however, certain models can only be specified in one framework or the other because of program limitations. For example, inherently (fully) nonlinear models can only be directly fit within the (nonlinear) multilevel modeling framework, and second-order growth models can only be fit within the structural equation modeling framework. Furthermore, some models are more easily fit within a certain framework, although these models can be fit in both frameworks. For example, fitting growth models to data where individuals vary in their timing metric (individually varying time metrics) are more easily fit in the multilevel modeling framework, even though such models can be fit in the structural equation modeling framework (not necessarily with all structural equation modeling programs). Similarly, growth models with mixture distribution and growth models with different residual structures are more easily specified in the structural equation modeling framework even though certain multilevel modeling programs

allow mixture distributions (e.g., PROC NLMIXED) and different residual structures (e.g., PROC MIXED; see Kwok, West, & Green, 2007). Thus, when moving into more advanced models, experience working in *both* the multilevel and structural equation modeling frameworks is beneficial.

As we noted, we discuss the programming of growth models using Mplus and OpenMx in the structural equation modeling framework and using PROC NLMIXED and nlme in the multilevel modeling framework. Mplus is a comprehensive latent variable modeling program (it can handle multilevel data, mixture distributions, and a variety of non-normal data [e.g., binary, ordinal, categorical, count, zero-inflated]), has efficient estimation routines (e.g., maximum likelihood, weighted least squares, Bayesian), a straightforward programming language, and is continually being improved. At the time of writing, Mplus is probably the most utilized structural equation modeling program. The Mplus website (www.statmodel.com) contains a demonstration version of the program that is only limited by the number of variables included in the analysis, the user manual, a collection of examples, discussion forums, and a series of papers highlighting new features of the program.

OpenMx can be seen as a recent update to Mx (Neale, Boker, Xie, & Maes, 2003), a freely available stand-alone structural equation modeling program. However, OpenMx is more of a transformation than an update because of the magnitude of its capabilities and how it is embedded within R, a freely available comprehensive statistical package. Thus, OpenMx is a free comprehensive structural equation modeling program that can handle binary and ordinal outcomes and mixture distributions. There are a variety of ways to specify models using OpenMx (path specification using RAM notation and matrix specification), but we note that regardless of the approach, the programming of OpenMx is more intense than Mplus, and familiarity with the R statistical package is beneficial. The OpenMx website (http://openmx.psyc.virginia.edu) contains program documentation, programming examples, a wiki, and forums where questions can be posed to the developers. Finally, the OpenMx development team is continuing to expand and improve its capabilities.

PROC MIXED and NLMIXED in SAS are two of the most popular procedures for mixed-effects or multilevel models. Singer (1998) provides an excellent overview of PROC MIXED, which increased its use among educational and psychological researchers. NLMIXED is a general modeling program that can handle multilevel data structures. Because of its generality, NLMIXED is not as efficient as MIXED; however, NLMIXED can handle inherently (fully) nonlinear models, non-normal outcomes (e.g., binary, ordinal, count, zero-inflated), and mixture distributions—topics that are of interest here. Additionally, the programming of NLMIXED is straightforward, although some knowledge of the SAS statistical language is beneficial.

The nlme package has been the primary mixed-effects modeling package available through R and includes both a linear mixed-effects modeling procedure (lme) and a nonlinear mixed-effects modeling procedure (nlme)—similar to MIXED and NLMIXED in SAS. Throughout this book we discuss the nlme procedure (over the lme procedure) because of its ability to fit inherently (fully) nonlinear models. The lme4 package (Bates,

Mächler, & Bolker, 2015; Bates, Mächler, Bolker, & Walker, 2011) is a newer package for fitting linear and nonlinear mixed-effects models (procedures include lmer and nlmer) in R and is able to fit mixed-effects models to non-normal outcomes (an advantage over nlme); however, nlme is more flexible when it comes to fitting inherently nonlinear models and its programming is more straightforward. For these reasons we focus on

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